Measuring Voice, Speech, and Swallowing in the Clinic and Laboratory
Measuring Voice, Speech, and Swallowing in the Clinic and Laboratory

Christy L. Ludlow
Raymond D. Kent
Lincoln C. Gray
## Contents

**Preface** vii

1 Signal and System Properties 1
2 Basics of Electricity 21
3 Principles of Measurement 39
4 Muscle Systems of the Aerodigestive Tract 57
5 Electrophysiology and Electromyography 91
6 Acoustic Recordings of Speech and Other Sounds 133
7 Acoustic Analysis of Speech 161
8 Measurement of Perceptual Attributes and Latent Traits 219
9 Visualizing or Imaging the Vocal (Aerodigestive) Tract 251
10 Measuring Movement During Voice, Speech, and Swallowing 289
11 Aerodynamics and Pulmonic Function 331
12 Measuring Voice Disorders 365
13 Measuring Swallowing Disorders 409
14 Using Stimulation to Assess and Treat Voice, Speech, and Swallowing 455
15 Measuring Brain Structure and Function with Neuroimaging 495

*Appendix A. International System of Units* 527
*Appendix B. Conversions of Measurement Units* 529
*Appendix C. Standardized Reading Passages* 531
*Appendix D. Disinfection Standards for Clinical Devices* 539
*Index* 541
Measurements that are valid, accurate, and reliable are essential to the assessment and treatment of persons affected by disorders of speech, voice, and swallowing. Many measures are made with instruments that extend and complement the human senses. The effective use of instruments requires understanding of the basic principles underlying their operation, methods of calibration, protocols to follow in their use in the clinic and laboratory, availability of comparison data, and guidelines for interpreting results. Many different instruments are available, and the goal of this book is to provide basic information on the instruments and measures commonly used for assessing and treating persons with disorders of speech, voice, and swallowing for clinical practice, research studies, and conducting clinical trials.

Essentially, a measurement is a collection of quantitative data that characterizes an event, object, or function. Ideally, measurements are made with a standard unit and with standardized methods to ensure comparability of data across individuals, across times of measurement, and across settings such as different clinics. Measurements derived from instruments usually are made on some signal that is generated by a person or specialized equipment measuring a person.

This book covers several types of signals, including acoustic, electrical, magnetic, kinematic, optical, radiographic, and aerodynamic. Each type of signal is associated with procedures for data collection and analysis that are described in the relevant chapters. The first three chapters introduce signal properties (Chapter 1), electricity (Chapter 2), and the principles of measurement (Chapter 3) and lay the groundwork for the rest of the book. Chapter 4 is on the aerodigestive muscle systems essential to voice, speech, and swallowing, whereas Chapter 5 is on nervous system control, electrophysiology, and electromyography. The next three chapters are on making speech recordings (Chapter 6), acoustic analysis of speech and voice (Chapter 7), and perceptual methods of measuring voice and speech (Chapter 8). The next two chapters cover imaging the vocal tract and upper airway (Chapter 9) and measuring movements during voice, speech, and swallowing (Chapter 10). The next two chapters focus on assessing disorders and measuring treatment outcomes for patient care and clinical trials on voice disorders (Chapter 12) and on swallowing disorders (Chapter 13). The final two chapters are on using electrical and magnetic central and peripheral nervous system stimulation for the assessment and treatment of disorders (Chapter 14) and on neuroimaging techniques for the measurement of brain function and structure (Chapter 15). Although the scope of topics covered is relatively broad, it is by no means all inclusive. The authors have focused on what they consider of current importance and relatively new developments in the field.

The data of interest are obtained from living persons—patients seen clinically and subjects enrolled in research studies. Throughout this book, a premium is placed on minimizing risk and discomfort to the persons involved. Also important is the integrity of the data, which can be assured by following calibration routines, adhering to standards of data collection, and considering factors that could corrupt the signals of interest. By following
recommended procedures, the clinician and clinical scientist can obtain data of high quality without causing harm or discomfort to the patients or subjects under study.

Max Planck (1949) asserted, “An experiment is a question which science poses to Nature, and a measurement is the recording of Nature’s answer.” The same idea applies to clinical practice and clinical research. Clinical inquiry seeks to understand the nature and severity of a disorder, which are revealed through suitable measurements. These measurements help to give an understanding of pathology and an appreciation of how the pathology affects the lives of people. Much of the progress in medicine and its allied fields is the result of instruments that enable measurements of function. Instruments and measurements go hand in hand, and this idea resonates through the chapters of this book. An instrument is only as useful as the data it generates, and measures must be understood in terms of how the measuring instrument relates to body functions. By good use of the tools we have available today, we hope that we can better serve those we care for now and in the future.

Christy L. Ludlow
Raymond D. Kent
Lincoln C. Gray

Reference

The authors appreciate the generous sharing of knowledge and inspiration they have received from their mentors, colleagues and students over the last half century that was the basis for this book.
The purpose of this chapter is to set forth basic ideas about signals and systems. Much of this book is given to discussions of various types of signals and the systems that generate or process these signals. Both signals and systems have properties that determine the appropriate and allowable analyses with respect to instrumentation and the use of certain kinds of mathematics. Some of the most important properties are considered in this chapter, which is a general background for many concepts in the other chapters. Some readers may find this chapter challenging and perhaps a bit tedious because of its technical content and some rather abstract ideas. But mathematics is used modestly, and real-life examples illustrate the more difficult concepts. Although it is not essential to read this introductory chapter before reading other chapters, the information given here introduces terminology and concepts that undergird pretty much everything that follows. Laboratory methods depend on signals and systems. We begin with a discussion of signals.

**Signals**

**Definition of a Signal**

A signal can be defined in different ways. For example, it can be defined as a function representing a variable that contains information about the operation of a natural or artificial system. The word *function* means a mathematical operation performed on a variable of interest. If our interest is an acoustic signal, we can express the signal as \( y = f(t) \), where \( y \) is a value of sound amplitude and \( t \) is a particular point in time. Signal information can take many forms, such as a sound wave, a recording of muscle activity, the airflow from the nose or mouth, or a recording of a variable that reflects regional blood flow in the brain.

A signal also can be defined as a physical quantity that varies with time, space, or any other independent variable or variables. This latter definition is well suited to this book, as we will consider signals of many kinds, including acoustic, aerodynamic, visual, electrical, and electromagnetic. Many of the signals to be discussed in subsequent chapters pertain to physical quantities that change over time. Examples are the acoustic signal of speech, the electrical activity recorded from a muscle, and the flow of air from the mouth or nose. But we also will consider some signals that vary with space, such as images derived from methods such as x-rays.

The definition of signal that we prefer is a hybrid of the two definitions just given, that is, *a signal is defined as a physical quantity that varies with time or some other independent variable(s) and contains information about a system or phenomenon*. We are interested in signals precisely because they carry information, and this book is mostly about gathering and interpreting information about speech, voice, and swallowing. A signal often can be represented by a mathematical function, which is key to analysis possibilities. As we will see, a mathematical expression can take different forms, some of them based on equations and some on probabilities. Only a modest background in mathematics is assumed in this chapter.
Signal Transduction

Frequently, the information of interest is in terms of a signal that is converted from the original signal to a form of energy that is more convenient for the purposes of storage or analysis. A transducer is a device that changes one form of energy into another. The most common type of converted energy is an electrical signal, which is discussed in Chapter 2. A familiar type of transducer is a microphone, which converts sound energy into electrical energy. Most of the signals discussed in this book are converted to electrical energy so they can be stored, processed, and analyzed efficiently. Several forms of energy (mechanical, optical, acoustic, thermal, chemical, and hydraulic) can be converted to electrical energy. Signal transduction is so common that it is nearly ubiquitous. But that does not mean that it can be taken for granted with respect to its fidelity. As various types of signal transduction are considered in this book, we will examine risks to fidelity and how to guard against them. Basic aspects of signal transduction are discussed further in Chapters 2 and 3, and the topic of transducers will recur frequently in many chapters as different measurement systems are considered. Transducers are mentioned briefly here because they are the means to almost every signal discussed in this book. It is well to keep in mind that a transducer stands behind the signals we seek to measure and analyze.

Transducers must satisfy four general criteria. First, they should be accurate, maintaining fidelity with the original signal that was transduced. Second, they should be stable (see Chapter 3 for a discussion of precision related to stability). Third, they should not interfere with the variable under observation or measurement. For example, a transducer should not have physical properties such as mass or strain that affect the tissues on which it is placed. And, fourth, they should not pose discomfort or harm to the individual who is being examined or studied. Fortunately, miniaturization of transducers has helped appreciably to make them adaptable for many purposes and to minimize discomfort and risk. These criteria are general guidelines for the design and selection of transducers for biomedical and related purposes. The basic work of a transducer, then, is to give us a signal that can be used to study or assess a given phenomenon. Signals are of several types, as discussed in a later section.

Transducers are classified as active or passive, depending on whether an external power source is required for their operation. Active transducers do not require a power source, because they work on the principle of energy conversion to produce an electrical signal that is proportional to the physical property of interest. For example, a thermocouple is an active transducer used to measure temperature. It can be constructed from two legs of different metals, and temperature is measured at their junction point. Passive transducers require a power source and typically work by generating an output signal of varying voltage or current. For example, a photocell is a passive transducer that is sensitive to the amount of light that falls on it but depends on a power source to generate an output signal.

The term sensor sometimes is used interchangeably with transducer, but the two are not really synonymous. To understand the difference, let us consider a microphone and a speaker. Because a transducer is a device that can convert energy in any direction, both the microphone and the speaker are transducers. However, only the microphone is designed to function as a sensor in its capability to pick up signals that can control electrical or electronic circuits. In contrast, at least in ordinary conditions, a speaker does not create signals that are used for such a purpose. We can shout at a speaker all day but not exercise control over a circuit. For the purposes of this book, a sensor is defined as a mechanical device sensitive to a particular physical property (such as heat, light, sound, pressure, magnetism, or motion).
that transmits a signal to a measuring or controlling instrument. All the sensors discussed in this book are transducers; that is, they act by converting one form of energy to another.

**Signal Types**

Signals are classified in several ways, but we will focus our attention on certain categories that are most relevant to speech, voice, and swallowing.

**Continuous Time and Discrete Time Signals**

A **continuous signal** is one that is defined for all instants of time. The sine wave shown in Figure 1–1a is an example. For any point on the time axis (and there is a potentially infinite number of such points, depending on how finely we divide the time axis), there is a corresponding value of the signal amplitude (and there is a potentially infinite number of amplitude values, depending on how finely the measurement is made). A **discrete time signal** can be derived from a continuous time signal if samples are taken at discrete points in time, as illustrated in Figure 1–1. In examples 4 and 5 above, the difference is whether the sound energy is monitored continuously or discretely. Many naturally occurring phenomena are associated with continuous signals, and our sensory systems are stimulated by an ever-changing combination of acoustic, thermal, and electromagnetic energy. Whenever we sample these phenomena at discrete intervals, we are dealing with data in discrete time.

**Analog and Digital Signals**

The word analog as applied to signals is similar to **continuous**, as defined above. Some definitions treat these two terms as synonymous. Other definitions draw a distinction, often a rather subtle one. One definition is that an **analog signal** is a continuous signal containing time-varying quantities. Another is that an
analog signal is any continuous signal for which the time-varying feature (variable) of the signal represents some other time-varying quantity. In telecommunications, an analog signal is one in which a base carrier’s alternating current frequency is modified in some way, for example, by amplifying the strength of the signal (amplitude modulation, or AM) or by varying the frequency (frequency modulation, FM). The modulation adds information to the signal. Broadcast and telephone transmission conventionally are based on analog technology. What is common across the various definitions of the analog signal is that the signal has the attributes of a continuous signal as defined earlier.

A digital signal is very much like the discrete time signal defined in the discussion of continuous versus discrete time signals. A digital signal is discrete in both time and amplitude. Much more will be said about digital signals in forthcoming chapters. The major point to be made in this introductory chapter is that a digital signal is discrete in time because the signal is sampled at regular intervals called the **sampling rate** and it is discrete in amplitude because this dimension is **quantized**, or represented in small increments (quanta). This is illustrated in Figure 1–2, which shows how a sine wave is converted to a digital form by sampling at regular intervals and then quantizing the amplitude variations. The result, as shown in part B of the figure, is a sequence of rectangles or pulses that represents the original continuous waveform. The digital version is essentially a sequence of narrow rectangles that vary in amplitude. The quantization of amplitude is expressed as the number of bits used in amplitude coding. With each additional bit of amplitude conversion, there is a doubling of levels of quantization, for example,

<table>
<thead>
<tr>
<th>Bits</th>
<th>Levels</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>256</td>
</tr>
<tr>
<td>9</td>
<td>512</td>
</tr>
<tr>
<td>10</td>
<td>1,024</td>
</tr>
</tbody>
</table>

The greater the number of bits, the finer the gradation of signal amplitudes, and therefore, the better the fidelity. Digital signals are very important because they are the essence of digital signal processing, about which we will have much more to say in ensuing chapters. Chapter 3 builds on these basic ideas, and other chapters discuss their importance to issues such as audio recordings (Chapter 6) and acoustic analysis (Chapter 7).

It is sometimes said that we live in an analog world. The input signals to our sensory systems are analog (continuous) at their source. This is true of sounds, smells, photos, images, and videos. The world comes to us in a panoply of continuous signals. We are analog creatures in terms of our basic physiology.
Air, blood, and other fluids move about our bodies in a continuous fashion. Our muscles contract and relax in a continuous pattern. But increasingly, the world we live in is a hybrid world that is both analog and digital. The digital world is largely one of our own creation. As digital computing has swept through more and more of our daily lives, we have come to depend on digital signals and the digital systems that process these signals.

**Stationary and Nonstationary Signals**

All signals can be categorized as either stationary or nonstationary. A signal is said to be stationary if its long-term statistics do not change. A sinusoid that is continued indefinitely is a stationary signal because it has the same characteristics at any given moment of inspection. A continuous random noise also is said to be stationary because it has stable long-term statistics. Nonstationary signals have variations in their long-term statistics. Speech is a common example of a nonstationary signal because its amplitude and spectral properties change rapidly as different sounds are produced. A major challenge in studying nonstationary signals is the need to take account of how the signals change over time. Speech carries information precisely because its signal varies in complex ways over time. Accordingly, the analysis of speech needs to be done in a way that is sensitive to these time-wise variations. The same is true for recordings of movements, electrical activity in muscles or neural tissues, and movement of air into and out of the lungs.

**Deterministic and Nondeterministic Signals**

A signal is deterministic if it is exactly predictable for the time interval of interest, that is, there is no uncertainty about its value. A sinusoid is both stationary and deterministic. Random noise is stationary but nondeterministic. Deterministic signals can be described by mathematical models. A signal is nondeterministic if it is not exactly predictable over the time interval of interest, that is, there is uncertainty about its value at some instant of time. Nondeterministic signals are also called random or stochastic signals. Such signals cannot be expressed by a mathematical formula but can be modeled in probabilistic terms. The word stochastic is derived from the Greek stochastikos, meaning “skilled at aiming” (stochos is a target). This sense of the word carries the challenge in analyzing nondeterministic signals. Many signals considered in this book have both deterministic and nondeterministic components, that is, they have a highly predictable component (signal of interest) that is mixed with an unpredictable component (noise).

**Periodic and Aperiodic Signals**

A signal is periodic if it satisfies the condition

\[ x(t) = x(t + T_0) \]

where

\[ T_0 = \text{fundamental time period} \]

\[ 1/T_0 = f_0 = \text{fundamental frequency} \]

A periodic signal repeats itself at every interval \( T_0 \). A notational convention is to express the fundamental period as \( T_0 \) and the fundamental frequency as \( f_0 \) and this convention is followed in this book. For example, when we refer to the fundamental frequency of vocal fold vibration, we will use the symbol \( f_0 \). Knowing that a signal is periodic opens the door to the use of very powerful mathematical operations. An important example of a periodic signal is illustrated in Figure 1–1a: a sinusoid repeats at the interval of its fundamental period and is therefore completely predictable during its
duration. An aperiodic signal does not repeat itself at a regular time interval. Random noise is an example of an aperiodic signal. Such a signal does not repeat itself at any interval and is therefore unpredictable.

The definition of periodicity just given extends to a digital signal, where a number of samples \( n \) is involved:

\[
x(n) = x(n + N)
\]

where

\[ N = \text{number of samples in a fundamental period} \]
\[ 1/N = f_0 = \text{fundamental frequency} \]

Whether a signal is continuous or digital, the notion of periodicity applies. Knowing that a signal is periodic is extremely important in its identification and prediction.

**Chaotic Signals**

As if things were not complicated already, we need to consider signals that have a new set of properties. A chaotic signal is defined as being deterministic and aperiodic and presenting sensitivity to initial conditions. The first two properties have already been defined; the last property means that, if the system generating the signal starts with a slightly different initial condition, the signal quickly diverges from the original one. How and when a system begins can greatly influence its eventual state. Chaotic signals are best understood in terms of chaos theory, which is discussed later in this chapter when we turn to different kinds of systems. Chaotic signals are being increasingly recognized in processes related to speech and voice. Although the word chaos connotes disorder, chaotic signals and systems can be analyzed, as described later. Although they may appear noise-like, they have more structure than noise, and it is the detection of this structure that is both challenging and important.

**Energy and Power Signals**

An energy signal has a finite energy \( (E) \), that is, \( 0 < E < \infty \). Energy signals have values only within a limited time interval. For example, a square pulse is an energy signal. The power of an energy signal is 0 (dividing finite energy by infinite time). A power signal has finite power \( (P) \), that is, \( 0 < P < \infty \). A power signal has an infinite duration. An example is a sine wave with an infinite length. The energy of a power signal is infinite, which is of no practical value.

**Signals: An Example from Voice**

As an example of the signals to be covered in detail in this book, let us examine four types of voice waveforms that have been suggested to have clinical utility in the study of voice disorders. These waveforms are shown in Figure 1–3. The waveform labeled A is from a normal voice. It is not strictly periodic but very nearly so. Notice the regularity with which the basic pattern is repeated, having a \( T_0 \) of about 5 ms (there are about two glottal cycles in the first 10 ms). This waveform is essentially periodic and deterministic. With the waveform labeled B, it is still possible to detect periodicity, but we have to look a bit harder, as there is considerable cycle-to-cycle variation. The waveform labeled C is basically aperiodic, but some structure can be seen that at least hints of periodicity. Finally, the waveform labeled D is aperiodic and can be described as stochastic noise. Procedures for the analysis of voice are discussed in Chapter 12; for now, the point to be made from Figure 1–3 is that the signals of voice and speech can take different forms that affect the choice of appropriate analysis methods. For example,
1. Signal and System Properties

an analysis suited to waveform A would not necessarily be appropriate to waveform D.

Signals in the Time and Frequency Domains

In most applications, signals are represented in two major ways: time domain and frequency domain. The time domain can be expressed in either continuous (analog) or discrete (digital) terms:

\[ y = f(t) \]—analog

\[ y = f(n) \]—digital

The most commonly used frequency domain representations are the Fourier transform and the Laplace transform. The Fourier transform (discussed in more detail in later chapters) decomposes a time function (e.g., a sound wave) or an image (e.g., a medical image) into its frequency components (a spectrum). Specifically, it decomposes a signal into a linear combination of sine and cosine components. This transform is widely used and certainly qualifies as one of the most important mathematical transforms ever developed. For our purposes in this book, the Fourier transform is the means of computing the spectrum of a signal, including sound waves, images, and electrical recordings from biological tissues. Examples of Fourier analysis for waveforms are shown in Figure 1–4. Note that periodic sounds are associated with regularity in the time domain and discrete energy locations in the frequency domain. Periodicity in the time domain implies discrete locations of energy in the frequency domain.

Figure 1–3. Waveforms of four of the voice samples used in the analysis. A–D. Samples of type 1, 2, 3, and 4 voices, respectively. The waveforms in panels A and D are from female voices, whereas those in panels B and C are from male voices. Reprinted with permission from A. Sprecher, A. Olszewski, J. Jiang, et al., Updating signal typing in voice: addition of type 4 signals, Journal of the Acoustical Society of America, 127, p. 3712. Copyright 2010, Acoustical Society of America.
In contrast, the aperiodic nature of random noise is associated with a wide spread of energy in the frequency domain. Fourier analyses are featured repeatedly in this book. The Laplace transform is a special transform that works very well in solving differential equations, in particular, differential equations describing mechanical systems and electric circuits. It is central to many applications in engineering and is used to analyze a quadrature signal, that is, a signal having a real and an imaginary part that is a function of time. Such a signal is a complex number. A single complex number can be represented by a point on the two-dimensional complex plane, also called the S plane. Such a plane has two axes (real and imaginary) that are orthogonal to each other.

Another important transform, the Z-transform, produces still another kind of representation by converting a discrete time signal into a complex frequency domain representation. It is essentially a discrete time equivalent of the Laplace transform.

Laplace and Z-transforms are very important in engineering applications, but we will not say much more about them in this book. Readers who delve more deeply in digital signal processing likely will encounter these transforms again.

**Signals of Primary Interest in this Book**

Many different types of signals are involved in the study of speech, voice, and swallowing, and these signals are the means to several methods of clinical assessment and treatment. Signals discussed in this book include the following:

1. Acoustic (audio) signals (Chapters 6 and 7)
2. Aerodynamic signals (Chapter 11)
3. Electrical and electromagnetic signals (Chapters 2, 5, 10, 11, 14, and 15)
4. Mechanical signals (Chapters 9, 10, and 11)
5. Optical signals (Chapters 10 and 15)

In these forthcoming chapters, we encounter signal processing as the transformation of signals for efficient manipulation, transmission, or storage. For the most part, the signal processing will be digital rather than analog. But it should be kept in mind that most signals of interest arise in nature as analog signals, and the conversion to digital form must be done with due consideration of the principles of analog-to-digital conversion (ADC).

**Systems**

**Definition of a System**

A system can be defined as a group of interacting, interrelated, or interdependent elements
forming a complex whole. This definition applies to very different phenomena, for example, the solar system, an automobile transmission, an electrical circuit, the central nervous system, or the muscular system of speech production. Another definition of a system is a device or set of computations that performs an operation on a signal. In many cases, the system receives a signal as input and generates a signal as an output. As will be seen, some of the properties described so far for signals also are used in describing systems.

Systems can be enormously complicated, but our discussion begins with a very simple block diagram that will serve as a basic template for system description. This diagram, shown in Figure 1–5, shows the system being controlled as a box commonly called a plant that receives an input signal and produces an output signal. The system performance can be graphed as the relationship between input and output, as illustrated in Figure 1–5. This very general characterization of a system is elaborated on later in this chapter.

Systems are classified in several ways, some of which are described next.

Continuous Time Systems

In continuous time systems, the input and output signals are continuous. Examples are sensors such as microphones, electrodes, and the sensory receptors of the human body. These systems receive a continuous input and provide a continuous output. They are also called analog systems. A continuous system can be expressed mathematically in the form of a differential equation. This equation states how a rate of change (a “differential”) in one variable is related to other variables.

Discrete Time Systems

In these systems, both the input and output signals are discrete. They are expressed mathematically in the form of a time difference equation. A difference equation expresses a value of a sequence as a function of the other terms in the sequence.

Linear Systems

A linear system exhibits the property of additive superposition (sometimes called simply superposition), meaning that if the system is excited with an input that results in an output \( A \) and then is excited with an input \( b \) that results in an output \( B \), excitation of that system with input \( (a + b) \) will result in the output \( (A + B) \). This property can be expressed mathematically as

\[
F(a) + F(b) = F(a + b).
\]

Linearity is basic to many mathematical operations and is therefore very important in signal and system analysis. Even when a system is nonlinear, it may have a piecewise linearity in its performance, and that allows analysis based on the assumption of linearity for the linear segment.
Time Invariant Systems

The property of time invariance means a system behaves in the same way for any two trials in time so long as the inputs and starting conditions are the same. This property can be expressed mathematically as follows:

If an input signal \( x(t) \) produces an output \( y(t) \), then any time-shifted input, \( x(t + \delta) \), results in a time-shifted output \( y(t + \delta) \).

In other words, advancing (or delaying) the input to a time invariant system results in the same advance (or delay) of its output. If a system is time invariant, we know that it will behave in the same way over time given the same inputs and starting conditions.

Linear Time Invariant (LTI) Systems

Systems that have the joint properties of linearity and time invariance comprise an important class of systems for which powerful mathematics (e.g., Fourier transform) can be used. If these properties cannot be assumed, it is necessary to consider other mathematical approaches. We will consider examples in later chapters.

Chaotic Systems

Chaos theory investigates strange behavior found in nonlinear deterministic dynamical systems. The words nonlinear and deterministic in this phrase carry the same meanings as discussed earlier. Dynamical systems are physical phenomena (or mathematical objects used to model these phenomena) whose state or instantaneous description changes over time. Many examples can be seen around us: smoke rising from a chimney, water flowing in a bubbling stream, clouds interspersed with areas of clear sky, and a strong wind blowing dust. Many systems in the human body are best characterized as dynamic because their functional properties vary over time.

Poincaré often is credited as the first to note unusual behaviors in dynamical systems. He was studying the solar system in the late 1800s and observed that in his mathematical models, small differences in initial conditions could lead to very different final solutions. He wrote in an essay in 1903 that “it may happen that small differences in the initial conditions produce very great ones in the final phenomena.” Several decades later, Lorenz, in developing a system of coupled nonlinear differential equations to model weather patterns, also noted the sensitivity to initial conditions. He called this sensitivity “the butterfly effect” to suggest that the flapping of a butterfly’s wings in one region of the world could influence the weather in distant parts of the world. Today, even those who know little about the scientific study of chaos are familiar with this effect, so that the butterfly effect is part of our cultural lexicon, aided no doubt by the appearance of the term in movies such as Jurassic Park and The Butterfly Effect. The present discussion will be more on the prosaic side and without the dinosaurs that populated Jurassic Park and its sequel(s).

It may seem peculiar that a system can be deterministic but not immediately predictable. The reason why these ostensibly contradictory notions can be reconciled is that dynamical systems can be described in terms of trajectories in a phase space, which is an abstraction in which the system is replaced by a representation of the space of possible states that the system can assume. In other words, dynamical systems are mathematical objects used to model physical phenomena whose state (or instantaneous description) changes over time. For example, depending on air temperature, water can take the form of a liquid, solid, or vapor. These are distinct states. Or consider locomotion in a horse, which can