

# **Making Mathematics Accessible for Elementary Students Who Struggle**

Using CRA/CSA for Interventions



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# Preface

The three authors of this book are former teachers and current university faculty who prepare teachers and conduct research in the area of mathematics instruction for students with and without disabilities. Our areas of expertise are different since we come from elementary education and special education, but our purpose in writing this book is to provide teachers with information about learning trajectories and strategies for teaching students who struggle in mathematics. There is much overlap of ideas across our fields, but differences in terminology and approaches may sometimes overshadow our common purposes. This book was born out of many conversations, collaborative activities, and research among the authors about effective mathematics interventions. Students who struggle may have disabilities or may be students who lack prerequisite understanding and skills. These students' intervention teachers may have general education backgrounds or special education backgrounds, and both need information about intervention practices from general education and special education. It is the authors' intent to provide teachers with approaches and methods that are of sufficient intensity to support students but also guide students toward independence. Each chapter shows how the concrete-representational/semi-concrete-abstract (CRA/CSA) sequence is used to support student learning across elementary mathematics concepts. The use of objects (concrete level), pictures, and drawings (representational/semi-concrete) prior to instruction using numbers only (abstract) will provide students with experiences that build conceptual understanding and then lead students to procedural knowledge and fluency. It is important that students are able to explain their thinking and reasoning at each phase of this process and to see the connections between the different representations of content that are learned through CRA/CSA. Learning mathematics using CRA/CSA will allow students to develop conceptual, procedural, and declarative knowledge that will be used to engage in mathematical practices.



Teacher presents large group of counters and asks students how this might be made into smaller equal groups.



The group of 34 counters is separated into groups of five.



Once separated, skip count the groups and use shortened counting to arrive at total.

5      10      15      20      25      30      31, 32, 33, 34



Write addition equation representing the counting process.

$$5 + 5 + 5 + 5 + 5 + 5 + 4 = 34$$

**Figure 3–17.** Sets of Objects With Shortened Counting and Equation.

tion, students can create drawings in journals or the teacher can use pictures that represent quantities larger than 20. An example is shown in Figure 3–18.

First the teacher reviews why people skip count and explains that skip counting is an easier way to count larger quantities. She shows students a journal or a picture with nonsymbolic amounts larger than 20 and models circling equal groups. She then skip counts the groups to determine the total amount and writes an equation that represents what she did. Once she models skip counting, she asks students to count quantities with her. She displays another amount larger than 20 using a journal or picture and



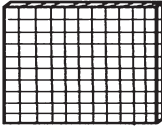
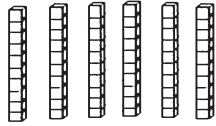

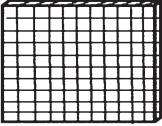
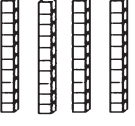

solved using drawings within a place value chart. Simple drawings represent ones (short tallies), tens (long vertical lines), and hundreds (squares). An example of using partial sums to solve an addition problem at the concrete and representational/semi-concrete level is shown in Figure 4–10.

Research has shown that CRA/CSA is effective in teaching students who struggle with mathematics addition with the traditional algorithm associated with regrouping (Miller & Kaffar, 2011). The CRA/CSA sequence supports the development of conceptual knowledge of the operation. Within the curriculum materials developed as a result of this research, problems are solved at the concrete level using base 10 blocks, which are organized using a place

**Problem:** There are two different choices on the school lunch menu, hamburgers and tacos.

Three hundred students brought their lunch from home and the others chose to eat a school lunch. Of the students who ate at school lunch, 164 students chose to eat a hamburger and 147 students chose to eat tacos. How many students chose to eat a school lunch?

**Think Aloud:** *The problem asks about how many students ate a school lunch. There is information about students who brought their lunch and students who ate school lunches. We do not need to know about students with lunches from home, only those who ate a school lunch. There are two kinds of students who ate school lunch. Some ate hamburgers and others ate tacos. Altogether, they ate school lunch, but this number is unknown. This is a part-part-whole problem. We know the parts, hamburger and taco eaters, but not the whole. The parts are joined together to make the whole. When groups are joined, the operation is addition.*

$164 = 100 + 60 + 4$ $+147 = 100 + 40 + 7$		
Hundreds	Tens	Ones
		
		

**Figure 4–10.** Solving Addition With Regrouping Using Partial Sums at the Concrete and Representational Levels. continues

<p>There are 195 students in the fourth grade. There are 15 students in each fourth-grade classroom. How many fourth-grade classrooms are there?</p> <p><i>The problem asks how many classrooms or groups of students. The fourth-grade students are grouped into classrooms with 15 students in each. The total number, 195 students, is separated into groups of 15. When a number is separated into groups of the same size, the operation is division.</i></p> <p>195 ÷ 15 = is how many groups?  <small>separated into groups of</small>      <small>many groups?</small></p>	
<p>Using partial quotients, find how many groups of fifteen can be made. The process of separation begins in the hundreds place. The hundred is broken into tens. Now there 19 tens. There are 15 tens (10 x 15) within 195. So, I note ten to the side. I subtract 150 from 195. Now, observe how many groups of 15 can be made with the 45 that are left. Three groups of 15 can be made. Add the partial quotients to find the answer. Add 10 and 3 for an answer of 13.</p>	
$\begin{array}{r} 15 \overline{)195} \\ - 150 \\ \hline 45 \end{array}$	
<p>Now, observe how many groups of 15 can be made with the 45 that are left. Three groups of 15 can be made (break a ten into ten ones to make groups of 15). Add the partial quotients to find the answer. Add 10 and 3 for an answer of 13.</p>	
$\begin{array}{r} 15 \overline{)195} \\ - 150 \\ \hline 45 \\ - 45 \\ \hline 0 \end{array}$ <p style="text-align: right; margin-right: 20px;">10 3 13</p>	

**Figure 7–7.** Concrete-Level Instruction in Division Using Partial Quotients.

since the ÷ symbol is replaced with the half-rectangle encased around the dividend, (b) problem solving begins in the left-hand column rather than in the ones place on the right, and (c) the separation of the dividend can become lost with an emphasis on multiplication in finding the quotient. Teaching

### Model Equivalent Fractions Using Area Model

**Present problem.** *I have three sixths and it is equal to a fraction with a denominator of 2.*

3	=	
6		2

**Make completed fraction.** *First, I am going to make three sixths. I need to find the blocks that have six parts in the whole. The numerator tells how many of these sixths I need. What is the numerator? Yes, 3. So I put three sixths together.*



**Make equivalent fraction.** *I am going to find the fraction that is equal. The denominator is 2, so I need to find the blocks that have two parts in the whole. The numerator tells how many I need. So, I find the one half blocks and see how many it takes to equal three sixths. It takes one, so the numerator is 1. I compare the two to make sure that they are the same.*



**Figure 9–5.** Making Equivalent Fractions at Concrete Level. continues